Overview

The Plasma Science and Innovation Center (PSI-Center) is refining and developing codes to accurately simulate innovative confinement concept (ICC) experiments, with the goal of achieving predictive capability. The PSI-Center development focuses on geometries, boundary conditions, and atomic physics, plus two-fluid, non-local parallel heat flow closures, and kinetic effects, appropriate for ICCs. Simulations are performed in collaboration with twelve ICC experiments at seven institutions, and are compared with experimental data. The results of these comparisons guide further code development and refinement for improved predictability. This Progress Report covers the period 1 Mar 2005 – 31 Dec 2009.

Introduction

The Plasma Science and Innovation Center (PSI-Center) is a U.S. Department of Energy-funded program to improve computational predictability of innovative confinement concept (ICC) experiments. The PSI-Center is adding the necessary physics and algorithmic capability to extended magnetohydrodynamic (MHD) codes, and is working with experimentalists to validate code results with experimental data. The computational groups work closely with the experimentalists to iterate this process, summarized in this Mission Statement: “In concert with smaller innovative plasma physics experiments, refine and optimize existing MHD codes to achieve significantly improved predictive capability.” The PSI-Center emphasizes physics that may extend beyond the standard analysis presently applied to the mainline (e.g., tokamak) fusion programs. This specifically includes strong flow effects, kinetic effects, reconnection and relaxation phenomena, transport, atomic physics, radiation, FLR effects, two-fluid or Hall physics, more realistic boundary conditions and geometry, and other physics that must be included in models to achieve the needed predictability. All of these effects are also important in mainline fusion devices, but one or more tend to dominate effects in particular ICC
configurations, which makes those effects particularly amenable to ICC study with their existing diagnostics. The goal of the PSI-Center is to capture the dominant effects of many different ICC experiments, covering most of ICC physics, and is shown schematically in the following flow chart.

![Flow Chart]

**PSI-Center Organization**

The PSI-Center is based at the University of Washington, Seattle WA, the University of Wisconsin-Madison, Madison WI, and Utah State University, Logan UT, and organized in four groups (codes and methods mentioned in this section will be described in detail in subsequent sections):

- **Admin. (Pareja-Klemisch)**
- **Deputy (Milroy)**
- **Director (Jarboe)**

*Group Lead

<table>
<thead>
<tr>
<th>Group</th>
<th>Lead</th>
<th>Institution</th>
</tr>
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<tbody>
<tr>
<td>Boundary Conditions and Geometry</td>
<td>Shumlak*</td>
<td>Wisconsi</td>
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<td>MarklinS</td>
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<td>Sovinec*W</td>
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<td>Kim'</td>
<td></td>
</tr>
<tr>
<td>Interfacing Group</td>
<td>Nelson*</td>
<td></td>
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* Group Lead
W at Wisconsin
U at Utah
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N presently at NRL
The **Boundary Conditions and Geometry Group** works on more realistic geometries, boundary conditions, atomic physics, neutrals, and radiation. They work on the development of the HiFi code and also a 3D tetrahedral mesh MHD code, PSI-TET, to accurately model the complex geometry of many ICC experiments.

The **Two-Fluid, MHD Transport, and Relaxation Group** works on two-fluid/Hall, relaxation/reconnection, and MHD transport. They develop capabilities for modeling 3D dynamics in plasmas that are either collisional or in transition to low-collisionality behavior. These tasks build upon the mature MHD algorithms and Hall-MHD algorithms in the NIMROD code to produce self-consistent two-fluid simulation models for the lower-temperature plasmas found in ICC experiments.

The **FLR and Kinetic Effects Group** works on kinetic and FLR effects, primarily focusing on adding the physics needed for modeling field-reversed configurations (FRCs) with 3D codes. Energetic particles are simulated using a $\delta f$ particle-in-cell (PIC) model. Time-dependant boundary conditions have been implemented in NIMROD to accurately simulate the external coils used in theta-pinpin formation, translation, compression, and RMF current drive experiments.

The **Interfacing Group** works with the ICC experiments, and the other three PSI-Center Physics Groups. They lead the PSI-Center's interactions with the participating experimental groups. In coordination with the collaborating experiments, the best applicable code and equations to be solved are selected. Interfaces to code output are developed, and continually refined for data presentation and analysis packages. The Interfacing Group performs ICC experiment outreach and information dissemination through the PSI-Center web site, and presentations at conferences and workshops.

**PSI-Center Code Development**

The PSI-Center is concentrating its efforts on two major extended MHD codes, NIMROD and HiFi, as well as other developmental/testing codes.

**NIMROD:**

The Non Ideal Magnetohydrodynamics with Rotation, Open Discussion code, NIMROD,$^{1,2}$ is a macroscopic simulation code that solves compressible nonlinear magneto-fluid equations with electric-field terms selected to represent either the non-ideal single-fluid MHD model or two-fluid models of magnetized plasmas. The NIMROD code is 3D, using spectral finite elements to represent a non-periodic plane and finite Fourier series for the periodic direction. The finite elements provide a flexible description of the poloidal plane, so NIMROD is suitable for a variety of ICC configurations. The code also has flexibility in the polynomial degree of the basis functions, so that spatial convergence can be achieved through the most efficient combination of mesh resolution and basis function order. High-order basis functions have proven important for simulating the extremely anisotropic thermal transport associated with magnetized plasma$^1$, particularly when the magnetic field is not aligned with the computational mesh, as is common in simulations of dynamic ICC experiments. It is also important for distinguishing parallel and perpendicular forces and for maintaining small numerical divergence error in the magnetic field.
The most important kinetic effects for macroscopic dynamics result from particle mean-free-paths being large in plasma of moderate to high temperature. The NIMROD Team (the USU group, in particular) has analytically and computationally developed integro-differential and continuum velocity-space methods for closing fluid moment equations with parallel-kinetic effects\(^3,4\). The USU group has also developed a very general hierarchy of moment equation\(^5,6,7,8\) for incorporating kinetic effects, as discussed below. The kinetic computations include quantitative particle-collision operators and are valid over a range from short-mean-free paths to collisionless behavior. This is important for modeling ICC experiments where there is strong interaction between confined high-temperature plasma and cooler edge plasma.

NIMROD also models kinetic effects from energetic minority ions with a \(\delta f\) PIC method. A \(\delta f\) pressure moment is calculated and added to the MHD momentum equation. This minimal modification to the MHD equations is valid in the limit of \(n_{\text{HOT}} << n_{\text{BULK}}\) but with \(\beta_{\text{HOT}} \sim \beta_{\text{BULK}}\)\(^9,10,11\).

With NIMROD, the PSI-Center was able to start from a working code that had been developed, benchmarked, and used by the NIMROD Team (http://nimrodteam.org). The Center has enhanced NIMROD to improve modeling capabilities and performance for ICC experiments in the following ways:

- Prior numerical analysis work indicates that NIMROD’s implicit leapfrog algorithm for two-fluid computation\(^7\) requires implicit advection. With PSI-Center support, we implemented the changes that are necessary for nonlinear computations in each of the field-advances. This implementation has since been used for a number of two-fluid applications in 2D and 3D.

- Nonlinear artificial particle and heat diffusivities were implemented to improve performance in conditions of strong flow. The NIMROD algorithm has very little inherent numerical dissipation, which allows accurate simulation of moderate and high temperature plasma, and truncation errors are predominantly dispersive. However, flow with large gradients, which occurs in simulations of translating toroids, suffers from overshoot errors when the dispersion is not damped. To correct the problem, we implemented nonlinear anisotropic diffusivities that are proportional to the relative change induced by flow during each time step. For number density \((n)\), for example, this added particle flux is

\[
\mathbf{f} = -f \left( \frac{\Delta t \mathbf{V} \cdot \nabla n}{n} \right)^2 \left( \frac{A_e}{\Delta t} \right) \mathbf{V} \mathbf{V} \cdot \nabla n
\]

where \(\mathbf{V}\) is the flow velocity, \(A_e\) is the element area, \(\Delta t\) is the time-step, and \(f\) is a numerical factor less than unity. This enhanced flux acts like numerical upwinding and is only active where there is advection of large gradients; it also vanishes in the limit of high resolution. For PSI-Center applications, it has been used successfully in FRC translation simulations.

- As first shown in Ref. 1, high-order finite elements are important for numerical convergence of thermal conduction in the extremely anisotropic conditions of magnetically confined plasma. Recent work for the PSI-Center finds that convergence can be further improved through application of a mixed finite element...
method, where an expansion for the parallel heat flux is solved simultaneously with temperature. A comparison of convergence properties with the standard high-order method and with the mixed finite element method is shown in Figure 1. Although more variables are solved simultaneously with the mixed method, the significantly reduced resolution is cost effective in NIMROD simulations, where other fields are solved and where this approach can reduce the number of kinetic computations for nonlocal transport (see below).

![Figure 1](image)

**Figure 1** Error in effective perpendicular thermal diffusivity as a function of mesh spacing for bicubic, biquartic, and biquintic elements using the standard method and for bicubic elements using the mixed method (light blue).

- Motivated by the success of the mixed method described above, we are developing a direct, implicit, numerical solution of the drift kinetic equation (DKE) with coefficients for the kinetic distortion (non-Maxwellian part of the distribution function) solved simultaneously with temperature, $T$. The computational cost is much closer to calculations without any kinetic effects, and the approach can be generalized to include time-dependent kinetic effects, unlike our integro-differential approach. Figure 2 compares profiles of temperature using the $T$-only and the $T$ coupled to DKE approaches.
Figure 2 Steady-state temperature profiles show flattening across a magnetic island in periodic cylinder geometry. The blue curve and red curves show cases with polynomial degree of 2 and 5 using the standard T-advance. The green curve shows the result of solving the coupled temperature-DKE system with polynomial degree of 2.

- We have constructed sets of higher-order fluid moments equations with accurate, linearized collisional effects. In developing parallel closures for general collisionality, the accurate evaluation of the collision operator is a crucial step. The exact linearized Coulomb collision operator and its moments are calculated for arbitrary masses and temperatures in the total-velocity moment expansion. For electron-ion collisions, the random-velocity moments of collision operators are also calculated with the small mass-ratio approximation. An infinite hierarchy of general moment equations is built by taking moments of the Landau kinetic equation. We also have written explicit formulas to calculate nonlinear terms of Landau-Fokker-Planck operators for arbitrary mass and temperature ratios. By solving parallel moment equations, the integral forms of the parallel heat flow and viscous stress are obtained. The application of the integral form for the parallel heat flow in numerical studies of heat confinement in SSPX is described in the applications section below. We have also found that the inclusion of the ion-electron collisions in ion transport significantly alters the ion parallel closures for highly and moderately collisional regimes. It also affects perpendicular ion transport for most ranges of magnetic field strength. A convergence study shows that the coefficients of the perpendicular frictional force in the Braginskii closures should be checked with increasing the number of moments.

- We have developed fitting formulas for electron transport coefficients at arbitrary magnetization to 1% accuracy. The formulas for the heat flow and friction have a simple form and will be straightforward to implement. We have also developed formulas for viscosity coefficients, where we find that the errors in Braginskii's coefficients are less significant (<7%) than those in heat flow and friction.

- NIMROD’s fluid-based closures have been improved for better modeling of low-temperature plasma. The parallel ion stress, $\Pi_\parallel = (p_i \tau_i / 2) (\mathbf{b} \cdot \mathbf{W} \cdot \mathbf{b}) (I - 3 \mathbf{b} \mathbf{b})$, where $\mathbf{W}$ is the traceless rate-of-strain tensor and $\tau$ is the collision time, now has a
temperature-dependent coefficient. We tested the implementation on weakly damped waves and verified that the ion-acoustic and magneto-acoustic waves damp at correct rates and that the parallel Alfvén wave is not damped. We have also implemented and verified the magnetization factor for perpendicular thermal conduction. This is important for very low temperature regions, typically adjacent to walls, and limits perpendicular conduction as collision become more frequent than gyro-motion orbits.

- We have implemented the semi-empirical Chodura resistivity\textsuperscript{14} to model conditions where electrons are accelerated beyond significant inertial and drag responses. Here, electron microinstabilities are important, and their macroscopic effects have been described by a semi-empirical resistivity formula:

\[
\eta_{ch} = \frac{m_e V_{ch}}{n e^2}, \quad V_{ch} = C_v \omega_{pe} \left[ 1 - \exp \left( -f \frac{v_d}{v_s} \right) \right]
\]

where \(v_d\) is the electron drift velocity and \(v_s\) is the acoustic speed.

- Improvements in NIMROD’s boundary conditions include time-varying external coils for FRC formation, translation (as well as a wall radius that can vary along the Z-axis), rotating magnetic-field current drive, and end-shorting/Hall physics to capture FRC spin-up effects.

- We have optimized NIMROD to improve computational performance and parallel scaling for applications that require many FFT operations, which is the case in simulations of ICCs with strong relaxation. Array indices and loops for the finite-element integrals have been reordered to efficiently use fewer FFTs and associated collective communication operations with more data each time to reduce latency. Changes in numerical integration also make better use of cache. The improvements lead to practical parallel scaling to the ten-thousand-processor level, as shown in Figure 3.

![Figure 3](https://via.placeholder.com/150)

Figure 3 Weak scaling results performed on the Cray XT4 at NERSC after optimization of numerical integration in NIMROD. Traces show total ‘loop’ time and ‘FFT’ transform time (which includes the collective communication as toroidal resolution/decomposition is varied. Each color represents a different level of poloidal decomposition (blue=32 blocks, black=64 blocks, and red=128 blocks).
• An addition to NIMROD’s preprocessing is the ‘stitch’ code that is used to assemble sets of regularly ordered blocks into irregular shapes. It is useful for modeling experiments with more than one distinct region, such as spheromak experiments with distinct gun and conserver regions. It can also be used to connect logically rectangular mesh blocks into non-rectangular shapes (see Figure 4).

![Figure 4](image)

**Figure 4** Borders and labels of separate meshing regions (left) and final assembled mesh (right) for simulating the proposed spherical plasma dynamo experiment\(^{15}\) at the Univ. of Wisconsin.

**Hybrid Kinetic-MHD in NIMROD:**

Energetic particle simulations were performed to study the impact of increasing the energy cut off of the particles loaded into the simulation. It was observed in the drift kinetic hot particle simulations that the \(n=1\) projection of \(\delta f\) in velocity space peaks at the extreme values of velocity (see Figure 5). To follow up on this observation, simulations were performed where the velocity cut off was increased from 1x10\(^{6}\) m/s to (1.1, 1.2, 1.3, 1.5)x10\(^{6}\) m/s. It was observed that as the velocity cut off is increased, the relative amplitude of \(\delta f\) for passing region to trapped region decreases. Stabilization at \(\beta_{\text{frac}}=0.5\) was found to increase (almost linearly) with increasing velocity cut off, with a velocity cut off 1.5x10\(^{6}\) m/s entirely stabilizing the (1,1) mode. The real frequency was also observed to increase (linearly) with increasing velocity cutoff (see Figure 6). It is also worth mentioning that for the fixed \(\beta_{\text{frac}}\), as the velocity cut off is increased, the energetic particle density is decreased to maintain the constant particle pressure.

These simulations show the importance of the details of the energetic particle distribution. These results may be significant when applying the hybrid kinetic-MHD method to ICC devices such as FRCs and RFPs (particularly extensions of MST tearing mode simulations with realistic geometry).
Figure 5  $n=1$ projection of $\delta f$ in velocity space shows activity dominantly in the trapped region and extremes of the passing region.

Figure 6  Velocity cut off scan and impact on linear growth rate and real frequency of (1,1) mode for $\beta_{\text{par}}=0.5$. Note near linear dependence.

NIMROD now has full orbit Lorentz PIC particle pushing in a rectangular grid FRC. Special routines have been added for a uniform regular mesh that greatly improves the PIC performance. Using these particles, we have performed some particle tracing simulations to understand the equilibrium orbits of ions in an FRC configuration with 4.15 mWb of trapped flux. The higher energy ions have very unique orbits that lie outside the usual MHD regimes. Figure 7 shows a sequence of particle orbits, with each sub-figure showing a single orbit over many transit times. The FRC flux is shown as a transparent green surface, and two sets of magnetic field lines are colored by their magnitude (one set approximately the last closed field line and the other set the first open
field line). The particle orbit is colored by the toroidal velocity. By coloring the particles by their toroidal velocity the time evolution of the trajectory is obscured. However, another interpretation of the plot is to view the particles as a closed trajectory of a phase space plot in \((x, y, z, v_\theta)\). Figure 7 (a) shows a betatron orbit with an \(m=2\) structure, while Figure 7 (b) shows a different view of the same particle orbit. This particular betatron orbit exhibits an \(m=2\) lobe structure that shifts 90 degree as the orbit crosses the symmetry plane. Betatron orbits with larger “\(m\)-mode” have also been observed. Another example is the resonant orbit, Figure 7 (c). Note that the toroidal velocity is always positive. This is typical for most of the energetic population. For comparison, Figure 7 (d) shows a low energy orbit. The usual gyro-orbit is clear in the tight spirals. Also present but not quite as obvious is the drift motion of gyrocenter due to curvature and gradients in magnetic field. Beyond single particle traces we can also look at the equilibrium distribution function. We initialize the particle sample (millions of particles) as an isotropic Maxwellian. We push the particles many transit times until the distribution reaches a steady state.

Observation of the velocity space distribution shows a strong anisotropy in the \(v_\theta\) direction, with particles strongly preferring the co-rotating direction. This anisotropy is more pronounced as the temperature is increased. The velocity space distribution plots are shown in Figure 8. Figure 8 (a) and (c) show the initial isotropic loading for 100 eV, and 2.5 keV respectively. The resulting steady state distribution of particles after many transit times is shown in Figure 8 (b) and (d). For the 100 eV case, the distribution (plotted in \((v_\theta, |v_z|)\)) remains relatively symmetric. For the 2.5 keV case, the distribution evolves to a strongly co-rotating one.
Figure 7: Particle orbits for a single particle in an FRC field.
HiFi Code:

The PSI-Center is continuing to develop the 3D high order finite (spectral) element code framework HiFi, based on its previously developed 2D version also known as SEL\textsuperscript{16}. The distinguishing capabilities of the code include fully 3D adaptive spectral element spatial representation with flexible multi-block geometry (under development), highly parallelizable implicit time advance, and general flux-source form of the PDEs, and boundary conditions that can be implemented in its framework. The 2D SEL code has been extensively validated and used for simulations of various multi-fluid plasma physics phenomena, including magnetic reconnection, cylindrical tokamak sawtooth oscillations, and FRC translation. HiFi uses the hierarchical data format (HDF5) for parallel data I/O and a separate post-processing code for data analysis and generation of data files readable by the 3D visualization software VisIt (described later). HiFi can currently read single-block grids generated by the Sandia CUBIT meshing program (http://cubit.sandia.gov). Among recent additions to the physics modeling capabilities of HiFi are plasma-neutral interactions with a static neutral background, the Spitzer-Chodura resistivity model, the full Braginskii magnetic field dependent anisotropic heat conduction tensor, and the 3D Hall MHD model. Present development efforts include self-consistent boundary
condition formulations for open and insulated-conductor boundaries, extension to the
multi-block formulation, and implementation of physics-based preconditioning\textsuperscript{17} for
better parallel scalability. Additionally, HiFi has been and continues to be used for
studying fundamental numerical properties of the high order finite element spatial
discretization method. In particular, these include solving systems of PDEs with an
extreme degree of anisotropy\textsuperscript{18} and the numerical accuracy of computations on highly
distorted logically hexahedral structured grids.

**Scalable Parallel Solvers**

Parallel solution of a linear system is called scalable if simultaneously doubling the
number of dependent variables and the number of processors results in little or no
increase in the computation time to solution. Multiple length scales characteristic of
extended MHD lead to the need for high-order spatial representation and adaptive grids.
Multiple time scales lead to the need for implicit time steps and the resulting requirement
of efficient parallel solution of large, sparse linear systems. Scalability is essential for the
efficient use of current and future generations of parallel supercomputers with $10^4$-$10^5$
processors and petaflop speeds. Most extended MHD codes are not scalable, and as a
result are limited to a maximum of a few thousand processors that can be used efficiently.

We have developed a new method to achieve scalability for extended MHD modeling of
fusion plasmas. The heart of the approach is physics-based preconditioning, pioneered by
Luis Chacón.\textsuperscript{19} The physical dependent variables are partitioned into two sets. The first
set, e.g. density, pressure, and fields, is eliminated algebraically in terms of the second
set, e.g. momentum densities, reducing the order and condition number increasing the
diagonal dominance of the matrices to be solved. The resulting Schur complement matrix
is approximated and simplified by interchanging the order of substitution and spatial
discretization. This procedure leads to an effective preconditioner, which accelerates the
convergence of a final nonlinear Newton-Krylov solver on the full system of equations.
Approximations to the Schur complement affect only the rate of convergence, not the
accuracy of the final solution. Solution of the reduced matrices uses additive Schwarz
blockwise LU-preconditioned FGMRES, with many variations available through the
PETSc library. The code is organized into a general-purpose solver and a separate,
problem-dependent application module, which can adapt the method to any system of
flux-source equations.

Weak parallel scaling tests on the NERSC Franklin Cray XT-4 computer up to 2,048
cores have shown partial scalability up for wall time and memory usage for visco-
resistive MHD, based on the well-known GEM Challenge magnetic reconnection
problem.
These tests show that the approximate Schur complement remains accurate and runs faster than the full static condensation approach, but the underlying solution procedure, based on additive Schwarz preconditioned FGMRES, requires improvement. One of the key ideas of physics-based preconditioning is that it assures the diagonal dominance of the reduced matrices, making it suitable for solution by algebraic multigrid. This will now be explored.

This work will be extended along the following lines:

- The algebraic multigrid library BoomerAMG, available though the Hypre and PETSc libraries, will be applied as the underlying scalable solver for the reduced matrices.
- Two-fluid effects will be incorporated
- The method will be ported to the 3D HiFi code, including multiblock domains.
- The method will be ported to other prominent extended-MHD macroscopic modeling code, including NIMROD and M3D-C1.

### Multi-Block Geometry Implementation in HiFi

HiFi/SEL now has the capability to handle multiple blocks in its computational domain. Each block (logical cube) is connected in a structured fashion. This allows for greater flexibility in the types of geometry that can be used for simulations, particularly non-simply connected and non-axisymmetric geometries. In addition to allowing more geometric possibilities, the development can significantly improve mesh quality when compared to a single logical block that would otherwise be severely distorted in order to achieve the same computational domain shape.

Figure 9 (a) shows an example mesh cutaway composed of five blocks of the ZaP Z-pinch geometry, and Figure 9 (b) shows a closeup of the detail at the cathode. This geometry configuration is possible with minimal mesh distortions due to the allowance of multiple blocks.
Figure 9: A three-dimensional cutaway of a 5 block ZaP z-pincher mesh (a) and zoom of cathode section (b) that is possible with the multi-block formulation of HiFi.

An unstructured multi-block development is underway and nearly complete. This allows for the logical blocks to be connected in an unstructured fashion. This will further improve the geometric capabilities, allowing for more complex domains while minimizing mesh distortions.

Other Developmental and Testing Codes:

In addition to NIMROD and HiFi the PSI-Center has other codes that perform more limited tasks. The most important of these is PSI-TET, the PSI center TETrahedral mesh mimetic operator code. This code computes multiple Taylor state eigenmodes and externally driven injector Taylor states on a tetrahedral mesh of arbitrary shape. It uses mimetic numerical operators which are constructed to exactly satisfy the relations div(curl)=0 and curl(grad)=0 for all variables. It has been used to compute Taylor states for HIT-SI and detailed comparisons have been made to probe measurements showing that the Taylor theory predictions are in excellent agreement with the experiment in both the spatial structure of the fields and the amplitude based on helicity balance with an empirically determined resistivity. In fact, out of all the PSI-Center codes, the Taylor state code has so far proven to be the most accurate tool available for predicting the behavior of this experiment. Taylor states have also been computed for SSX. PSI-TET can also compute 3D ideal MHD equilibria, using field line tracing to identify regions with good flux surfaces and solving a steady state parallel diffusion equation to compute a flux coordinate variable that is constant on the flux surfaces. The pressure and parallel current can be specified as arbitrary functions of this surface variable where surfaces exist and constants in ergodic regions where there are no surfaces. 3D equilibria with finite beta and hollow current profiles have been computed for HIT-SI. PSI-TET can also analyze ideal MHD stability of Taylor states or other ideal MHD equilibria. The center also has an axisymmetric ideal MHD equilibrium MATLAB code (Grad-Shafranov solver) that uses mimetic operators on a 2D triangle or quadrilateral mesh. The quad mesh version has been incorporated into the NIMROD suite and can be used for initialization.

Code Post-processing and Visualization:

VisIt (http://www.llnl.gov/visit/) is a powerful, open-source, 3D visualization code whose use is rapidly growing throughout the scientific community. It runs on many computer
platforms and can read a wide variety of data formats. It is run as client/server processes, where the server can be a remote machine, including running on many processors in parallel. Fast 3D OpenGL graphics are displayed on the client machine using the visualization toolkit (VTK) package. The HiFi post-processing generates HDF5 data which can be read via an easily-produced XDML file. A Python-based module “NimPy” can read standard NIMROD dumpfiles, as well as “nimplot”, and “nimfl” post-processor output, and produce VTK files that can be read by VisIt. NimPy also allows interactive viewing, plotting, and finite-element calculations of NIMROD data. This script-driven approach allows a standard NIMROD install to produce output without modifying/recompiling NIMROD. The PSI-TET code directly writes VTK files for post-processing in VisIt. This model of using powerful open-source software (and scripts to make input files for VisIt) provides a common platform across codes and experiments, easing the learning curve, and saving costly commercial 3D visualization licensing.

Computational Facilities

The NIMROD and HiFi codes make extensive use of the National Energy Research Supercomputer Center (NERSC http://www.nersc.gov), located at Lawrence Berkeley National Laboratory. At the University of Washington, the PSI-Center also has dedicated smaller local clusters and a firewall/data-serving network. The present cluster is a 16-processor 1.5 GHz Itanium2-based SGI Altix cluster, using NUMAl ink high-speed/low-latency interconnects. A secure gateway provides a firewall, web services, and SVN version control repositories for NIMROD, PSI-TET, and HiFi code development. A RAID6 data server provides file sharing for the cluster and workstations. Profs. Nelson and Shumlak were recently awarded an AFOSR DURIP equipment award to purchase a new 192-processor SGI Altix ICE 8200 2.8 GHz Xeon cluster with 384 GB memory, that will be jointly used by the WARP-X, ELF, and PSI-Center projects. This cluster arrived in Dec. 2008, and provides a powerful in-house development environment.

Collaborating ICC Experiments

The PSI-Center is actively collaborating with the following twelve ICC experiments at seven institutions: the Caltech Plasma Group21 (spheromak, helicity injection: Caltech), the field-reversed experiment FRX-L22 (FRC formation, translation, and active compression, Los Alamos National Laboratory), the steady inductive helicity injection spheromak HIT-SI23 (Univ of Wash: UW), the coaxial helicity injection (CHI) spherical torus HIT-II24 (UW), the levitated dipole experiment LDX25 (M.I.T.), the Madison symmetric torus MST26 (reversed field pinch, Univ of Wisc-Madison: UW-M), the Pegasus low aspect ratio spherical torus27 (UW-M), the pulsed high-density experiment PHD28 (FRC translation and compression, UW), the sustained spheromak physics experiment SSPX29 (coaxial helicity injection spheromak, Lawrence Livermore National Laboratory), the Swarthmore spheromak experiment SSX30 (dual, coaxial-source spheromak/FRC, Swarthmore College), the translation, confinement, and sustainment upgrade TCS-U31 (FRC translation and rotating magnetic field formation/sustainment, UW), and the ZaP flow-shear stabilized Z-pinches32 (UW).

Initial Results for Selected ICC Collaborations:

All four PSI-Center Groups are involved in simulations of the collaborating experiments. A brief description of initial results of some of these experiments follows.
**Generic FRC Calculations**

Several FRC studies have been performed which are not specific to any single experiment, but are of interest to several experimental concepts.

**FRC Tilt Stability Studies using NIMROD**

The tilt mode has been calculated to be the most dangerous global mode for FRCs. This mode is not readily observed in hot $\theta$-pinch formed FRCs, and this is generally attributed to two-fluid, FLR, and kinetic effects\(^{33}\) not captured by MHD. We have used NIMROD to examine the effect of the Hall term on the growth rate of the tilt mode. Ignoring the electron pressure term, the dispersion relation for the growth rate of the tilt mode for a uniform-long equilibrium\(^{34}\) has been obtained from Hall MHD

\[
\omega^2 + \frac{a\gamma_{\text{MHD}}}{S^*/E} \omega + \gamma_{\text{MHD}}^2 = 0,
\]

where the gradient of the electron pressure has been ignored. Solving for the Hall growth rate gives

\[
\gamma^2 = \gamma_{\text{MHD}}^2 \left[ 1 - \left( \frac{a}{S^*/E} \right) \right].
\]

The degree to which the Hall term affects the linear growth of the tilt mode is thus controlled by the ratio of the FRC elongation, $E$, to $S^*$, where $S^*$ is the ratio of the separatrix radius to the collisionless skin depth. $S^* = r_s/\delta$, $\delta = c/\omega_{pe}$. We can therefore vary this “kineticity” parameter by adjusting the initial density for a given elongation.

Results from NIMROD calculations are presented in Figure 10, which shows the square of the growth rate normalized to the MHD growth rate as a function of the parameter $E/S^*$. The growth is clearly suppressed by the presence of the Hall term and shows good linear scaling for modest values of $E/S^*$, but approaches a new regime at high values of $E/S^*$. This new regime is not yet well understood. Nonlinear simulations with several toroidal modes have also been performed and the results suggest some saturation of the $n=1$ tilt mode in the non-linear regime, but more studies at higher resolution are needed. These initial results have reproduced the earlier observed\(^{35}\) transition from the “fundamental” MHD internal tilt mode to modes with higher structure along $B$, with growth rates smaller but comparable to MHD growth rates.

This study has provided new insights into the effect that the Hall term can have on FRC stability, and it also served as an important benchmark on NIMROD’s Hall MHD capabilities.
Figure 10  The square of the normalized growth rate as a function of the “kineticity” parameter $E/S^*$. 

FRC Translation Benchmarks

We have successfully simulated the acceleration and translation of a Field Reversed Configuration (FRC) with the NIMROD code. In this simulation, an FRC was accelerated to super-sonic velocities by dynamic magnetic fields. Figure 11 shows pressure (solid colors) and magnetic flux lines for a calculation where an FRC is initially accelerated to the right. Here the FRC is initialized from an equilibrium code, and then $E_\theta$ is applied at the radial boundaries to simulate the firing of a compression coil over the left hand side of the FRC. This accelerates the FRC to the right where it reaches a peak velocity of about 1.5 times the acoustic velocity. It is reflected back to a lower field confinement region after colliding with a strong magnetic mirror at the end.
Codes typically have difficulty translating a configuration across a numerical mesh due to diffusion associated with the convective derivative. However, NIMROD, with its finite element implementation, has excellent conservation of FRC mass and trapped magnetic flux during the translation and capture process. These NIMROD calculations are only now possible due to recent modifications including the time-centered implicit advection algorithm, and an artificial diffusivity algorithm that reproduces an “upwinding-like” smoothing.

This calculation was run with a relatively small resistivity, and the trapped flux decayed to about 80% of its initial value, a remarkably small loss considering that a stationary FRC with this resistivity would have retained only approximately 85% of its initial flux. In addition, it was found that 85% of the initial particle inventory contained by the closed field-line region remained confined by the end of the simulation.

These calculations verify that the NIMROD code is an appropriate tool for numerical simulations of FRC experiments that involve a dynamic acceleration and translation as experienced in the FRX-L, PHD, and FRC thruster experiments.

End-shorting and FRC Spin-up

FRCs are usually observed to exhibit ion rotation shortly after formation, often leading to the $n=2$ rotational instability. The spin-up process is usually attributed to a combination of end-shorting and the loss of ions with a preferential angular momentum, which leaves the remaining confined ions with a net angular momentum in the opposite direction. This particle loss mechanism is not part of the extended MHD equations, but the end-shorting effect is captured if the Hall and $\nabla P_e$ terms are included, and the
The tangential component of the electric field is zeroed where the field lines intersect the wall. The basic physical mechanism is that if the generalized Ohm’s law is applied to a non-rotating FRC equilibrium, there is an axial gradient in the radial component of the electric field, especially near the ends where the $E_{\text{tang}} = 0$ boundary condition is applied. This leads to the generation of a toroidal field $B_\theta$, which in turn leads to a $J \times B$ torque which is applied to the plasma (an equal and opposite torque is applied to the conducting end-wall). The torque causes the open field-line plasma to spin-up, leading to a torque on the FRC due to a viscous drag.

This process has been simulated with the NIMROD code. The calculation is initialized with a non-rotating MHD equilibrium. The Hall and $\nabla P_e$ terms are included in the generalized Ohm’s law and $E_{\text{tang}} = 0$ is applied to the end boundaries. Figure 12 shows toroidal field and velocity that is generated on the open field lines after only 2 axial Alfvén times (an axial Alfvén time is about 25 $\mu$sec for the parameters used in this calculation). As time progresses, viscous coupling between the open field line plasma and the confined plasma in the FRC causes the FRC to gain angular momentum. The FRC spins up to its diamagnetic drift rate as illustrated in Figure 13, and in agreement with experimental observations. These calculations are an excellent example of the importance of including the Hall term in calculations that involve small plasma devices. A paper discussing these results in more detail has been published.

![Toroidal magnetic field, $B_\theta$ (top) and toroidal velocity, $V_\theta$ (bottom) at 50 $\mu$s (2 axial Alfvén times).](image)

Figure 12  Toroidal magnetic field, $B_\theta$ (top) and toroidal velocity, $V_\theta$ (bottom) at 50 $\mu$s (2 axial Alfvén times).
Figure 13  Toroidal velocity at the axial midplane as a function of radius at several times.

n=2 Stability studies

A study of the $n=2$ rotational instability, which is often observed in FRCs has been performed in collaboration with Loren Steinhauer\textsuperscript{39}. The $n=2$ rotational instability has almost always been observed in dynamically formed FRCs. This instability is driven by centrifugal forces in the rotating plasma, but it was found that it could be stabilized by the application relatively weak multipole fields. Translated FRCs often do not exhibit the $n=2$ rotational instability and recently Guo \textit{et al.}\textsuperscript{40} speculated that a small toroidal field that may be induced in the formation process of a translating FRC may be responsible for the stabilization of the mode.

Numerical simulations are initialized with an FRC equilibrium that has a weak toroidal magnetic field added with $rB_\theta = f(\psi)$. Here a simple linear dependence is assumed with $f \propto \psi$. Because of the relatively high elongation of an FRC, combined with the fact that in the inner region the separatrix reaches $r=0$, a relatively weak toroidal field can lead to a significant safety factor $q$. This is illustrated in Figure 14, which shows a single field line initiated at the axial midplane and at a radius of 0.1 $r_{\text{wall}}$, for a case where the peak toroidal field is 20% of the peak poloidal field strength. The magnetic surface upon which this field line lies has a $q \sim 1.3$. The field line path is similar to that of an elongated spherical tokamak.
Figure 14 Magnetic field line trace for a field line initiated at $z = 0$, and $r = 0.1r_{wall}$ assuming a peak toroidal field to peak poloidal field ratio of 0.2.

Full 3D numerical simulations of the growth of the $n=2$ rotational instability were performed using the NIMROD code. The Hall and $\nabla P_e$ terms are options that have been included in some of the simulations as will be noted. Numerical simulations are initialized with an MHD equilibrium, and then a rigid rotation and toroidal field is added. The resulting configuration is not quite in equilibrium but since NIMROD advances the full nonlinear equations, the solution rapidly evolves toward an equilibrium although some small $n=0$ oscillations appear. The initial FRC has an $x_s = r_s/r_{wall} = 0.65$, and an elongation $z_s/r_s = 5.8$. The dimensionless radius $S^* = r_s/l_i = 9.0$, and $E/S^* = 0.64$ ($l_i$ is the ion skin depth based on $n_{max}$). As a reference, past calculations showed that the Hall term reduces the growth rate for the $n=1$ tilt mode by about 50% for this value of $E/S^*$. An isotropic viscosity corresponding to a Reynolds number of about 400, and a uniform resistivity corresponding to a Lundquist number of about 8000 are assumed. An initial toroidal velocity $v_\theta = \Omega r$, with $\Omega$ chosen to yield a toroidal velocity approximately equal to the ion diamagnetic drift speed is assumed.

Figure 15 shows the growth in the $n=2$ kinetic energy as a function of time for the baseline parameters. The two upper (solid) curves are for the reference case with no toroidal magnetic field; the lower curves correspond to cases with successively higher peak toroidal field values of 10%, 20%, and 30% of the external $B_z$. The growth rate falls with increased toroidal field. For computations with peak toroidal field strengths of 20% or more, the onset of growth is delayed for several MHD growth times. This delay is an indication that the plasma is stable to a mode with a fixed displacement profile aligned with the initial velocity perturbation; the later emergence of growth indicates that the disturbance has evolved into one with a different structure that is unstable. Indeed, it was found that for increased toroidal field the unstable perturbation is confined to the open field line region and is especially concentrated to the region beyond. At times, complete stability has been observed experimentally, suggesting that not included in these simulations stabilize the open field line region. Perhaps a velocity shear in this region
that arises from end-shorting (not included in these calculations) stabilizes the open field line plasma.

Figure 15  Growth of the n=2 kinetic energy as a function of time for a series of calculations. Further details of this study can be found in Reference 39.

FRC Equilibrium Solver

A new flexible equilibrium solver, originally developed at the PSI-Center for the spheromak using mimetic operators, was extended for calculating FRC equilibria. The solver generates a solution to the Grad-Shafranov equation:

\[-\nabla^2 \psi = r^2 P'(\psi)\]

Assuming \(\psi=1\) at the null, pressure is specified as

\[P(\psi) = P_s + \frac{(1-P_s)}{1+\gamma}(\psi + \gamma\psi^2)\]

inside the FRC, and

\[P(\psi) = P_s e^{(\delta\psi + \alpha\psi^2)}\]

on the open field lines. The parameters \(\alpha, \delta,\) and \(\gamma\) are adjusted to achieve the desired FRC properties. To make \(P'\) continuous at the separatrix, the separatrix pressure \(P_s\) is set to

\[P_s = \frac{1}{1 + \delta(1 + \gamma)}.\]

Different pressure profile options can easily be added. This solver has proven to be very flexible and has allowed us to set initial conditions for a variety of FRC translation problems that can have a shaped boundary and wall flux that varies with axial position. An option to smoothly transition to a low pressure vacuum solution on open field lines far from the FRC has been included, which is important for simulations of experiments that employ a puff-fill and have a long translation region, such as PHD. An illustration of an equilibrium solution of an FRC with a shaped boundary is shown in Figure 16.
FRC 0-pinch Formation Studies

Historically, most FRCs were formed in a 0-pinch, and this method is also employed in present experiments including FRC-L, PHD, and optionally in TCSU. An FRC is formed in a 0-pinch when an initial reverse bias field is frozen into a cold pre-ionized plasma. A high voltage is then applied to the main 0-pinch coil, quickly reversing the current in it, and producing a closed field configuration. Two-dimensional MHD simulations of FRC formation have been performed routinely for many years with the Moqui code\textsuperscript{41}, and these results have been benchmarked against several experiments including TRX-1, TRX-2, FRX-B, FRX-C, LSX, and TRAP. We have begun similar calculations using the NIMROD code, with hope that its advanced algorithms will lead to more accurate simulations. NIMROD can more accurately handle the convective derivative, can better handle anisotropic transport, and can optionally include Hall and 2-fluid effects. Also, NIMROD can be run as a fully 3D code during formation to study stability issues at this time.

With the inclusion of Hall and 2-fluid effects, we can investigate the spontaneous generation of toroidal magnetic fields and flows during formation.

FRC formation employs the boundary condition modifications required to represent a series of discrete coils. An FRC formation calculation is illustrated in Figure 17, which shows the evolution of plasma density with magnetic field lines superimposed.
This calculation was initialized with cold plasma with a uniform density. An initial vacuum field (generated with the Matlab equilibrium solver) is applied with a negative bias under the central coil and a forward bias under the end coils. A high voltage (30 kV) is applied to the central coil quickly reversing the field under it, forming a closed field configuration, or an FRC. This FRC is far out of equilibrium, and undergoes a rapid axial contraction, and subsequent relaxation. These calculations employ a high resistivity to annihilate flux near experimentally observed rates ($\eta/\mu_0 = 250$ m$^2$/s) and a viscosity $\nu = 500$ m$^2$/s (Reynold’s number $\sim 200$).

A similar calculation, which employs the PHD method of simultaneously forming and accelerating the FRC is illustrated in Figure 18. Here the central coil is replaced with 3 discrete coils that are fired sequentially with a 1 µs interval, leading to an imbalance of axial forces that rapidly accelerates the FRC to the right.
A conically shaped $\theta$-pinch coil can also be employed to simultaneously form and accelerate an FRC. This method is employed in the FRX-L experiment, and a NIMROD calculation employing this method is illustrated in Figure 19. In this case the higher magnetic field at the small radius end of the coil leads to an axial force balance that accelerates the FRC toward the large radius end.
Figure 20 shows the time history of several key parameters from the conical $\theta$-pinch calculation. The trapped flux and particle inventory inside the FRC drops quite rapidly due to the assumed high resistivity. The temperature inside the FRC grows to a value 230 eV, but then starts to drop as the FRC expands adiabatically into the low field region to the right. All of these calculations were performed with the Hall and $\nabla P_e$ terms turned on, leading to the generation of significant toroidal magnetic field and plasma velocity. The “$B_\theta$ flux” in this figure is defined as $\int_{FRC} B_\theta dr dz$, and $|B_\theta$ flux| is defined as $\int_{FRC} |B_\theta| dr dz$.

Similarly the angular momentum $L = \int_{FRC} \rho r v_{\theta} 2\pi r dr dz$ and $|L| = \int_{FRC} \rho r |v_{\theta}| 2\pi r dr dz$. This figure shows that there is little net toroidal magnetic field induced, compared to the total “toroidal field activity”. This is in contrast to experimental observations. Future calculations will explore the possibility that a non-uniform resistivity, such as that induced with the Chodura formula, will lead to more net toroidal field, as was found in previous calculations.

**Figure 20 Time history of key parameters from simulation of FRC formation in a conical $\theta$-pinch.**

**HiFi Generic FRC Simulations:**

HiFi 2D calculations have been run for a translated FRC in a right cylinder, with a visco-resistive MHD equation set including Chodura/Spitzer resistivity and a static neutral background. The neutral background is depleted by electron-impact ionization (which adds to the ion mass), and also includes the effect of radiative recombination. The HiFi 2D runs (without neutrals) were verified against 2D NIMROD, then re-run with a neutral background.
Figure 21: Results from HiFi 2D simulations of FRC translation with a static neutral background. Upper, initial equilibrium, showing the four coil segments for translation (in proper aspect ratio); Lower, flux contours and neutral density color levels for 3 times in the simulation (expanded radial scale). The neutrals are ionized and incorporated into the ion fluid mass.

Figure 22: Comparison of HiFi 2D simulation results with and without a static neutral background. The ionization of the neutral fluid incorporates mass into the ion fluid, slowing down the FRC.

**Generic Spheromak Calculations**

The PSI-Center is using HiFi 3D two-fluid MHD capability to study fast relaxation of spheromak magnetic fields to the lowest energy state in a given closed perfectly conducting flux-conserver. The geometry under consideration closely resembles that of
both the older and newly-proposed SSX flux-conservers and leads to development of fully three-dimensional non-axisymmetric relaxed states.

Figure 23  Shown are B-field streamlines and pressure pseudocolor values at two different times. Left, mid-way through spheromak rotation and relaxation, and right is the final state. The initial condition is a constant lambda spheromak co-axial with the axis of the cylinder that constitutes the computational domain.

Experimental Studies of Spheromak Formation Physics, P. M. Bellan, (Caltech)

This experimental program investigates the physics of spheromak formation by using a magnetized planar coaxial helicity source. The design of this source has been optimized to have maximum geometric simplicity so that any complexities in observed plasma topology result from fundamental plasma dynamics and not from geometrical features of the source. The main issues being studied are topological evolution, helicity and mass injection, flows and stagnation, kink instabilities, flux amplification, relaxation and reconnection, and the generation of energetic particles.

The PSI-Center is presently running NIMROD simulations of the Caltech experiment. A suitably-packed rectangular grid was modified to allow poloidal current boundary conditions, \( v_i \), specifying \( RB_i(t) \) along the insulating gap. Hall-effect terms have been turned on and allow more parallel current to flow.
Figure 24: Mod B surfaces for NIMROD simulations of the Caltech experiment (w/o density evolution). Top left, initial dipole field; Top right, increasing current pushes the field out into the chamber; Bottom left, field reaches end of chamber; and Bottom right, current drives a kink instability.
Figure 25: VisIt generated field lines for an axisymmetric NIMROD simulation of the Caltech experiment. The applied current strongly twists the initially-poloidal fieldlines (shown as tubes with color denoting field strength) and pulls out the poloidal flux surfaces (seen as semi-transparent sheets).

Experimental Plasma Physics Programs at LANL, T. P. Intrator and G. A. Wurden, Los Alamos National Laboratory (FRX-L, FRCHX)

The P-24 Plasma Physics group is home to two ongoing plasma experiments that involve flows and non-time-stationary plasma physics in an essential way. FRX-L is a compact high energy density FRC that is translated into a liner compression region. FRCHX is an FRC confinement experiment.

Recent PSI-Center additions to the NIMROD and HiFi codes (external, independently-controllable conical theta-pinch coils) allow simulations of FRX-L formation and translation, as shown in Figure 17-Figure 19. A mimetic-operator-based Grad-Shafranov equilibrium solver allows solutions on NIMROD and HiFi grids, including transitions from regions of plasma to vacuum.

Helicity Injected Torus with Steady Inductive Helicity Injection, T. R. Jarboe, U. Wash (HIT-SI)

The HIT-SI experimental program investigates a concept to inductively drive current in a bow-tie spheromak plasma. It uses two non-axisymmetric injectors to inject helicity at a constant rate. The experiment has a complex 3D geometry that cannot be completely modeled with existing computational tools.
A Taylor-state solver has been used to calculate 3D eigenmodes of the HIT-SI geometry, as well as relaxed equilibria for the HIT-SI injectors. These equilibria agree remarkably well with experimental data, and have been published in *Physical Review Letters* 43.

Figure 26: Taylor-state calculations for HIT-SI at $\lambda=10.4$ m$^{-1}$: (a) current in one injector only ($I_{\text{inj}}$) without spheromak current ($I_{\text{tor}}$), (b) $I_{\text{tor}} \sim I_{\text{inj}}$, and (c) $I_{\text{tor}} \sim 5 I_{\text{inj}}$.

Figure 27: Comparison of Taylor-state calculation (dashed lines) with internal magnetic field probe data (crosses).

The PSI-Center has worked with HIT-SI personnel to implement the mimetic Grad-Shafranov solver on the HIT-SI NIMROD grid. As part of the multi-block work for HiFi, an unstructured group of structured blocks has been generated for HIT-SI using the CUBIT gridding software, as shown in Figure 28.
Figure 28: An example 3D mesh of a HIT-SI-like geometry showing one injector created by CUBIT is shown. The mesh is created with structured blocks of hexahedrons, where the connections between blocks are not necessarily structured. This enables the use of solvers that take advantage of structured mesh data structures, but also allows for non-simply connected geometries.

3D equilibria with variable lambda ($\lambda_{\text{INJ}} > \lambda_{\text{AXIS}}$) have been calculated for HIT-SI using the PSI-TET code, shown in Figure 29. Magnetic surfaces are found by solving parallel diffusion along field lines using an artificial temperature diffusion equation. Lambda is set to a function of $T$, and the resulting lambda profiles are constant on magnetic surfaces.

Figure 29: PSI-TET calculations of a HIT-SI lambda profile (left), and separatrix and open injector field lines for $\Phi_{\text{TOR}}/\Phi_{\text{INJ}} = 2$ (right).

Pulsed High Density Fusion Experiment, J. Slough, U. Wash. (PHD)

The PHD experiment forms an FRC in a large (0.4 m radius), high voltage 0-pinch chamber, and then accelerates and compresses the FRC through a series of small radius
sections. The goal is to compress and heat the FRC to fusion conditions, while remaining in a kinetic regime where the FRC is stable. The PSI-Centers addition of external coils, allows NIMROD to simulate the translation and compression of an FRC into a decreasing radius flux conserver. Figure 30 shows the assumed geometry and initial conditions for an initial simulation of PHD.

Figure 30 Initial density profile, with magnetic field lines superimposed for a simulation of the PHD experiment.

The calculation is initialized with an equilibrium, calculated with the Matlab equilibrium solver. The initial assumed plasma parameters include a peak temperature of \( T_e = T_i = 170 \text{ eV} \), and peak density of \( 2.6 \times 10^{20} \text{ m}^{-3} \). The large radius coils are fired sequentially with 30 kV applied at the simulation boundary, with a quarter cycle time \( t_{1/4} = 13 \mu s \), which leads to a 0.5 T swing in the vacuum field. A constant uniform resistivity \( \eta/\mu_n = 10 \text{ m}^2/\text{sec} \) (Lundquist number \( \sim 10^4 \)), and a constant uniform viscosity \( \nu = 500 \text{ m}^2/\text{sec} \) (Reynold’s number \( \sim 200 \)) was assumed. Figure 31 shows the evolution of density and magnetic field lines from this initial simulation of the PHD experiment. These calculations were performed with a numerical grid with 16 x 128 cells, employing 4\(^{th}\) order polynomials. The time history of some key parameters is shown in Figure 32.

Figure 31 Evolution of density and magnetic field lines for an initial simulation of PHD.
Figure 32  Time history of key parameters from PHD simulation.

Figure 32 illustrates good retention of trapped flux and mass during a significant translation. The FRC is accelerated to a velocity of over $3 \times 10^5$ m/s, and the temperature increases by a factor about 3.5, mostly due to adiabatic compression.

Future calculations will concentrate on the inclusion of more realistic transport (we will use a Chodura resistivity model) and the transport model will be tuned to match experimental observations. Formation calculations including the Hall term will be used to predict the generation of toroidal magnetic field resulting from the non-symmetric formation processes employed by PHD.

**Electrodeless Lorentz Force (ELF) Thruster, MSNW**

The PSI-Center (on a separate grant) performed simulations of the ELF project. ELF uses RMF in a conical theta-pinch to produce and eject an FRC plasmoid into an acceleration region. In the acceleration region, fields from an external coil set continuously produce a body force on the FRC as it transits through a background neutral gas. The FRC charge exchanges with the neutrals, with the “new” neutrals continuing with the previous velocity of the ion fluid, while the “new” slow ions are being accelerated. This continuous process increases the thrust mass to a much higher level than the initial FRC. Two-dimensional studies of a translating FRC interacting with background neutrals (through charge exchange, ionization, and recombination) in the HiFi (SEL) code are beginning. An initial FRC equilibrium ($20$ eV, $n_e = 10^{19}$ m$^{-3}$, $B_{\text{ext}} = 0.1$ T) in $10^{19}$ m$^{-3}$ peak density neutrals, Figure 33, is compressed and translated by an external coil, Figure 34.
Figure 33: SEL simulation of a translated FRC (white flux contours) in dynamic background neutrals (pseudocolor): Initial equilibrium.

The high-density compressed FRC ionizes the neutrals near $x=0$, and builds momentum density, transferring momentum to neutrals through charge-exchange.

Figure 34: SEL simulation of a translated FRC (white flux contours, black contours from translation coil), dynamic neutrals (pseudocolor), ion momentum density (white arrows), and neutral momentum density (black arrows). The FRC has been compressed, ionizing most of the neutrals around $Z=0$, and is transferring momentum to the neutrals at $Z > 0$.

The FRC is essentially stopped, having transferred its momentum to the neutrals through charge-exchange, and a sound wave is traveling to the right, Figure 35.
Figure 35: SEL simulation of a translated FRC (white flux contours, black contours from translation coil), dynamic background neutrals (pseudocolor), ion momentum density (white arrows), and neutral momentum density (black arrows). Near complete momentum transfer has occurred between the FRC and the neutrals.

Sustained Spheromak Physics Experiment, Bick Hooper, Lawrence Livermore National Laboratory (SSPX)

The SSPX experimental program investigated sustained confinement in a spheromak configuration. The experiment used a coaxial magnetized gun to inject plasma and helicity. Significant numerical simulation efforts have already been performed on this experiment. The program investigates relaxation flux surface quality, energy confinement, and boundary effects, among other phenomena.

PSI-Center efforts include use of our non-local, parallel heat flow closure in simulations\(^4\) of energy confinement of SSPX, which leads to improved agreement with experiment (see Figure 36). The dotted black curve is the result shown in C. R. Sovinec et al., PRL 94, 035003 (2005). The blue curve results from using the same transport model as the original reference, but with a mixed finite-element approach for the parallel heat flow. Improved spatial accuracy prevents parallel transport from polluting the perpendicular channel. The red curve shows a modest, additional improvement when using the non-local parallel heat flow closure. Future work will focus on higher temperature conditions, which were achieved in later SSPX discharges.
Figure 36: *Comparison of electron temperature measured with Thomson scattering in SSPX discharges Nos. 4620–4642 (boxes) with toroidally-averaged, NIMROD simulation results (lines). The dotted black curve shows the result from the original simulation. The blue curve results from using a mixed finite-element approach for the local, parallel heat flow closure. The red curve results from using the non-local closure.*

The PSI-Center-developed Python code suite “NimPy” has been transferred to LLNL personnel so they can convert NIMROD output to formats readable by the VisIt visualization software. Drs. Romero-Talamas and Hooper recently presented, at the 2008 ICC Workshop, very detailed analysis and insight from NIMROD simulations using NimPy and VisIt. These analyses are published in Ref. [45].
Figure 37: SSPX NIMROD data in VisIt: The semi-transparent slice in the y-z plane shows the structure of $|V \times B|$, where discrete bands of higher amplitude appear to approximately follow poloidal contours. Approximate magnetic field streamlines are shown with color corresponding to strength (these are not integrated in a finite-element sense). The red column in the middle represents a contour of constant current density. (Figure courtesy of Drs. Romero-Talamas and Hooper of LLNL, using the PSI-Center NimPy code to create VisIt input files.)

Swarthmore Spheromak Experiment, M. Brown, Swarthmore College and M. J. Schaffer, General Atomics (SSX)

The SSX/SSX-FRC experimental program investigates the merging of counter-helicity spheromaks to form an FRC, and the merging of co-helicity spheromaks to form a new spheromak. The program investigates magnetic reconnection during the merging, and stability of the resulting plasma configuration.

SSX Taylor-state equilibria have been calculated using the PSI-TET code, shown in Figure 38. Equilibria such have these have been successfully compared to SSX measurements of magnetic field structure\textsuperscript{46}.
Figure 38: SSX Taylor-state equilibrium calculated by the PSI-TET code.

The HiFi code has been used to model reconnection of two sphormaks in the SSX geometry, shown in Figure 39. The evolution of the magnetic field structure also agrees well with SSX magnetic field measurements.
Figure 39: 3D HiFi simulation of two spheromaks relaxing in an SSX-like geometry. Field lines are in gray, fluid velocity is in blue-to-red scale, and the current density is given by arrows in a rainbow color scale.

Translation, Confinement, & Sustainment Experiment Upgrade, A. Hoffman, U. Wash. (TCS-U)

The TCS-U experimental program explores using rotating magnetic fields (RMF) to form an FRC, as well as to build up flux, and sustain current in an FRC. The combination of RMF and neutral beam injection may provide a steady-state FRC. Important effects include the penetration of the RMF, the associated plasma flows, anomalous transport, relaxation, and stability of the resulting FRC. This process has been numerically modeled with a 2D (r-θ) MHD model, and more recently Belova has begun simulating RMF with the 3D HYM code.

A 3D extended-MHD study of RMF current drive has been performed with the NIMROD code, and the results have recently been submitted for publication. A module has been developed to optionally apply RMF boundary conditions in NIMROD. This is done by specifying a nonzero \( n=1 \) component to the tangential component of the electric field \( E_{\text{tang}} \), and the normal component of the magnetic field \( B_{\text{norm}} \) on the radial boundary. For finite length antennas, the \( E_{\text{tang}} \) has an \( n=1 \) \( E_z \) component with an appropriate axial shape function to center it at the axial midplane, and an \( n=1 \) \( E_\theta \) concentrated near the antenna ends to approximate the effects of the antenna return straps.
Simulations were performed assuming a straight cylinder with radius of 0.4 m, and a length of 7 m, extending from -3.5 m to 3.5 m. The 2 m RMF antenna was assumed to extend from -1 m to +1 m. The $n=1$ RMF field was assumed to ramp up from 0 to 5 mT over 100 μsec, and an RMF frequency of 117 kHz was assumed. The simulation volume was initialized with an $n=0$ axial vacuum magnetic field with strength of 4 mT at the midplane and increasing to 8 mT near the ends to produce a 2 x mirror in the end regions. The boundary flux for this $n=0$ field is held constant for the duration of the simulation. A uniform initial plasma density of $3 \times 10^{18}$ m$^{-3}$ was assumed. A constant and uniform electrical diffusivity $\eta/\mu_0 = 100$ m$^2$/s is specified, and a relatively large viscosity (Reynold’s number ~ 10) is required for numerical stability during the RMF formation process, however the viscosity can be reduced by a factor of ~100 after formation is complete, during the sustainment phase of a calculation.

Figure 40 shows the evolution of pressure contours during the formation phase of a typical 3D calculation that includes toroidal modes $n=0$ through $n=5$. It is noted that the FRC exhibits considerable distortion early, during the formation process, but later in time the distortions smooth out leaving a relatively smooth pressure profile.

Figure 41 shows a three-dimensional visualization of partially transparent pressure contours along with magnetic field lines late in time for the calculation shown in Figure 40. This is an even-parity calculation and, as expected, the RMF leads to an opening of

Figure 40: Pressure contours in an rz-slice at four times during formation. (Fig. 2 from Reference 4)

Figure 41: Three-dimensional image of partially transparent pressure contours along with magnetic field lines late in time for the calculation in Fig. 1. (Fig. 5 from Reference 4)
all of the field lines. The thick colored lines are launched from the axial midplane from a radius of less than 0.2 m. The thinner gray and yellow lines are launched from the end boundary and illustrate the field direction in the region beyond the ends of the FRC. Most of the field lines illustrated here exit the computational domain in the region under the RMF antennas, although some field lines transit from one end to the other, and some field lines enter the FRC from under the antenna and then exit the simulation region from an axial boundary. It is noted that the field lines beyond the ends of the FRC have been wound up, indicating that $n=0 \, B_\theta$ is generated in this region. This happens when the field lines penetrate the conducting wall end-plate where they are held stationary, while the boundary condition under the antenna dictates that they rotate at the RMF frequency. The strength of the resulting toroidal $B_\theta$ is influenced by the plasma resistivity and also by the boundary condition forcing $B_\theta = 0$ on the radial wall. This boundary condition is appropriate for an insulating boundary, but if in fact part of the wall is a metal conductor, it is not appropriate and a stronger $B_\theta$ is generated in that region. Some calculations were run with the assumption of a conducting wall and $B_\theta$ grew strong enough to drive an $n=1$ kink instability. This is a potentially important physical effect, but its strength will have to be tuned to measurements from individual experiments.

Antenna boundary conditions were then modified to permit the simulation of odd-parity antennas, rather than even-parity as assumed above. For odd-parity, the RMF is an odd function of $z$, but it still rotates the same direction on each side of the midplane. It has been found that under some conditions, field-line closure can be maintained with odd-parity RMF current drive\textsuperscript{51}. A calculation has been performed with similar parameters to those used for Figure 41, but with an odd-parity antenna and only modes 0 and 1 included. The field lines from late in this calculation are illustrated in Figure 42. This differs from the field lines illustrated in Figure 41 in several important ways. While many on the field lines are open, some of them are closed, or at least traverse the FRC many times. Field lines that enter under the RMF antenna region tend to exit at a $\theta$-value close to the $\theta$-value at which they entered, but on the other side of the axial midplane. For even-parity antennas, field lines tend to exit at a $\theta$-value approximately $\pi$ radians from the $\theta$-value at which they entered.

It has been suggested that RMF could be used to form an FRC using a slow formation mechanism, after which the RMF is rapidly switched off. It is important to understand

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig42.png}
\caption{Three-dimensional image of partially transparent pressure contours along with magnetic field lines late in time for an odd-parity calculation. (Fig. 8 from Reference 4)}
\end{figure}
the evolution of the magnetic topology as the RMF is quickly ramped off, after an FRC has formed. A key question is whether field lines that are open due to the RMF close, after it is switched off. This was investigated for both even-parity antennas and for odd-parity antennas. For the even-parity case shown in Figure 41, all of the field lines remain open after the RMF is ramped off in 10 µsec. While the RMF is on, most of the field lines enter and exit the radial wall under the antennas, but after it is turned off the field is tangential to the wall and they now enter and exit from the axial ends of the simulation region. For a similar calculation illustrated in Figure 42, but with odd-parity antennas, the field lines quickly close after the RMF is switched off. The FRC heals with either closed field lines or field lines which traverse the FRC many times. Clearly odd-parity is superior for this application.


The ZaP experimental program investigates the role of sheared flows on gross plasma stability in a simple Z-pinch geometry. The experiment couples a coaxial accelerator to a pinch assembly region to generate long-lived Z-pinch plasmas with embedded sheared axial flows.

The PSI-Center is presently examining sheared-flow stability on a periodic Z-Pinch. A “Bennett” Z-Pinch profile (polynomial in R) is loaded into NIMROD and a linear \( v_Z \) profile (constant shear) is imposed and the linear growth rates of the \( m=0 \) and 1 modes are calculated. Presently, a “Kadomtsev” profile, marginally stable to the \( m=0 \) instability, is being incorporated into NIMROD, to isolate stability boundary studies of non-axisymmetric modes.

Figure 43: Non-linear NIMROD simulation of sheared-flow stabilization of the Z-Pinch kink mode: Left, Evolution of the kink mode is evident without flow; Right, Kink mode is suppressed at same simulation time with a constant shear.

Levitated Dipole Experiment, Jay Kesner, MIT (LDX)

LDX produces a dipole magnetic field with a levitated superconducting coil. LDX uses RF power to produce plasma that is confined by the pure poloidal field. Plasma stability and confinement are studied.

The PSI-Center further refined a NIMROD LDX grid originally coded by Prof. Sovinec. (This new grid can be aligned with LDX pressure/flux surfaces.) An LDX pressure profile can be evolved onto the grid with a localized/shaped heating source, using only \( n=0 \) symmetry. Profiles evolved in this way may or may not satisfy the adiabatic
condition for interchange stability (depending on the source shape and intensity). The profile is then used to study higher order modes by running NIMROD with higher order $n$ terms enabled. (Difficulties in interpolating finite-volume equilibria onto the LDX finite-element mesh will be obviated with a native NIMROD grid finite-element equilibrium solver, which is being developed by another computational group. This solver will greatly aid stability analysis for marginally-stable adiabatic equilibria.)

Figure 44: Example of contours of pressure (larger set of contours) and the heating profile (smaller set of contours). Heat flows away from the source along the simple dipole field, produced by a current filament in the levitating coil (hole in the grid).

Figure 45: LDX NIMROD runs for a non-adiabatic (unstable) pressure profile showing contours of pressure (colortable), and velocity arrows, including toroidal modes $n=[0 .. 42]$. Left, initial condition; Right, after evolution of (predominantly) $n=6$ interchange.
These modifications to NIMROD have been given to Dr. Jay Kesner of M.I.T., and the PSI-Center will further assist Dr. Kesner and the LDX team in simulations.

**Madison Symmetric Torus, John Sarff, U. Wisc. (MST)**

MST is a reversed field pinch with minor radius 0.5 m, a relatively large experiment for the Center and is a proof of principle experiment. Collaboration with the Center has been to study the stabilizing effects of large orbit particles in the RFP. MST is an intermediate size experiment and will be invaluable in testing the Center’s codes in this regime.

Preliminary simulations of finite Larmor radius (FLR) effects on $m=1$ tearing instabilities in a reverse field pinch (RFP) plasma are presented. δf particle-in-cell (PIC) simulations were performed with the hybrid drift kinetic-MHD model to include the full Lorentz equations of motion to take into account FLR effects. These initial simulations show that for an idealized phase space distribution, sufficiently energetic ions stabilize the tearing mode (see Figure 46). These simulations show good agreement with analytic theory and demonstrate the FLR physics capability of the hybrid kinetic-MHD model.

![Figure 46](image_url)

**Figure 46** Growth Rate and Real Frequency vs. Perpendicular Velocity shows increasing Larmor radius decreases growth rate of tearing mode
Figure 47  *Tearing eigenmode with (right) and without (left) FLR effects shows broadening of mode structure and change of parity.*

It has been observed that the FLR effects strongly change the mode structure as shown in the Figure 47 & Figure 48. The figures show an eigenmodes of parallel flow for the case with and without FLR effects. Inclusion of FLR effects both broadens and changes the parity of the mode.

Figure 48  *Profile of eigenmode with and without FLR shows broadening and change in parity of mode*

The PSI-Center has also computed the first toroidal linear simulations of MST. Results of edge tearing mode simulation show good agreement with experimental measurements on MST, see Figure 49. These results have been included in an article submitted for publication by Tharp, Kim, *et al.*
Figure 49: Comparison of measurements of the \( m=0, n=1 \) mode in MST with cylindrical and toroidal calculations of the mode structure.

We continue collaboration with the MST group to understand the impact of energetic particles on the RFP plasma.
**Helicity Injected Torus II, Tom Jarboe, U. Wash. (HIT-II)**

HIT-II is a small spherical torus (ST) experiment used to study and develop the formation and sustainment of an ST by coaxial helicity injection (CHI). Modeling and understanding the observed flux amplification and closed-flux sustainment are the goals of this collaboration.

The PSI-Center has created a HIT-II NIMROD grid and modified boundary conditions to have two insulators (required for CHI). Simulations fit well when compared to data from non-relaxed HIT-II discharges\(^5^2\) and agree with plasma current scaling with the external toroidal field (Figure 50). A manuscript is in preparation for a journal article\(^5^3\).

![Figure 50: NIMROD simulation results and HIT-II experimental results of plasma current and injector current as a function of injector flux for conditions where helicity injection produces weak relaxation.](image-url)
Summary
The Plasma Science and Innovation Center (PSI-Center) focuses on improving computational predictability for innovative confinement concept (ICC) experiments. Three computational physics groups develop codes with physics and boundary conditions appropriate for ICCs. These groups and a fourth “interfacing” group collaborate with twelve ICC experiments at seven institutions on simulations of their experiments. Budget constraints make it difficult for most ICC experiments to pursue computations on their own, thus a modest, purposed effort such as the PSI-Center provides a synergistic path for improved understanding of the physics of ICCs. Improved predictability for ICCs provides a more direct and cost-effective path towards larger ICC-based experiments with improved chances for success. The PSI-Center has made significant progress towards these goals, both in code refinements for ICC experiments, and simulations of these experiments.

37 L.C. Steinhauer, “End-shorting and electric field in edge plasmas with application to field-reversed configurations”, *Phys Plasmas*, 9, 3851 (2002)